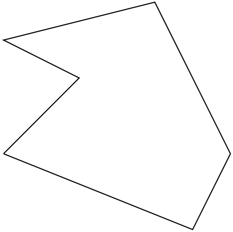
On the Travelling Salesperson Problem with Neighborhoods

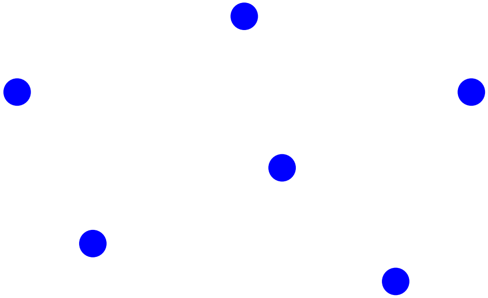
Antonios Antoniadis



June 28, 2019

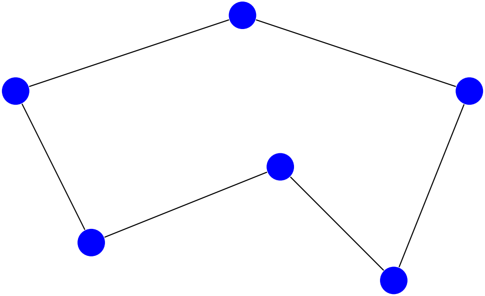
JPOC11

Travelling Salesperson Problem (TSP)



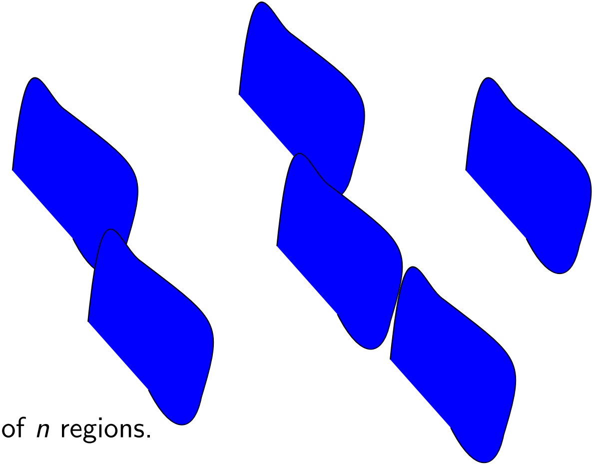
Input: A set of n points.

Travelling Salesperson Problem (TSP)



Input: A set of n points.
Output: A tour of minimum total length that visits all the points.

TSP with Neihgborhoods (TSPN)



Input: A set of n regions.

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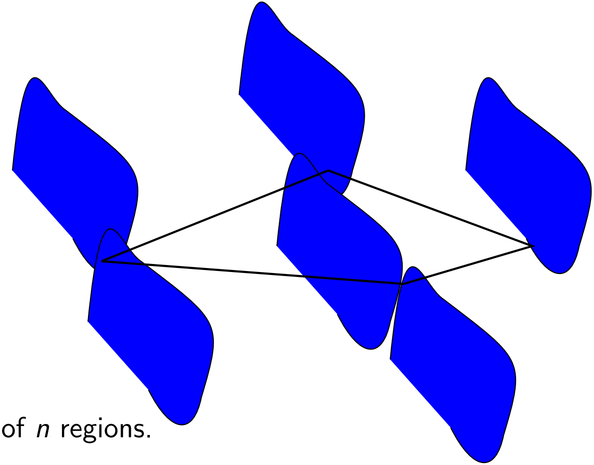

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Computational Complexity: TSP .vs. TSPN

	TSP	TSPN
exact solution	**NP-hard** Papadimitriou '77	**NP-hard** Papadimitriou '77
approximation	**Admits PTAS** Arora/Mitchell '96	**Does not admit PTAS** Gudmunsson & Levcopoulos '00

Definition

A polynomial-time algorithm ALG is called an α-approximation algorithm if cost(ALG,*I*)$\leq \alpha \cdot$cost(OPT,*I*) for all instances *I*

Definition

A PTAS is a family of algorithms $\{\text{ALG}_\epsilon\}_{\epsilon>0}$ such that for each $\epsilon > 0$, ALG_ϵ is a $(1+\epsilon)$-approximation algorithm.

Versions of TSPN

regions	**lower bound**	**upper bound**
k points (Group TSP) in $d = 2$	no const. apx. Safra & Schwarz '03	
$k = 2, d = 2$	no PTAS Dror & Orlin '03	
polygons in $d = 2$	no PTAS de Berg et al. '02	
conv. polytopes fixed d	NP-hard Papadimitriou '77	**Open Problem:** NP-hard? PTAS?
hyperplanes fixed d	**Open Problem:** NP-hard?	PTAS AA et al. '19
lines $d = 2$	-	$\in P$ Johnsson, '02
lines $d = 3$	NP-hard Papadimitriou '77	$\log^3 n$-apx. Dumitrescu& Tóth '13

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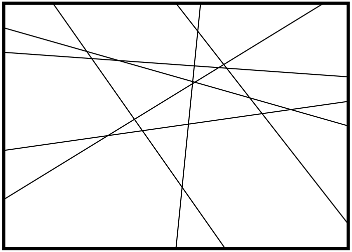
Hyperplane Neighborhoods, a Warmup

Dumitrescu and Tóth, SODA '13: a $2^{\Theta(d)}$-approximation



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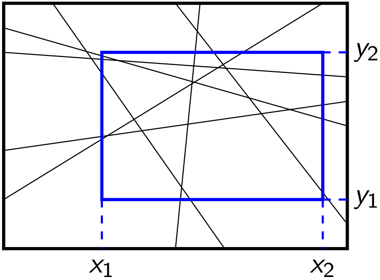

Linear Program:

$$\min (x_2 - x_1) + (y_2 - y_1)$$

$$\text{s.t.} \quad x_1 \leq x_2$$

$$y_1 \leq y_2$$

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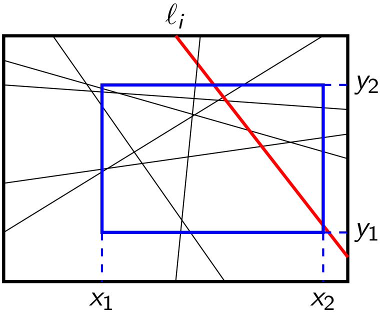

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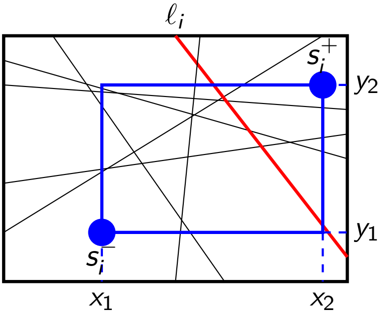

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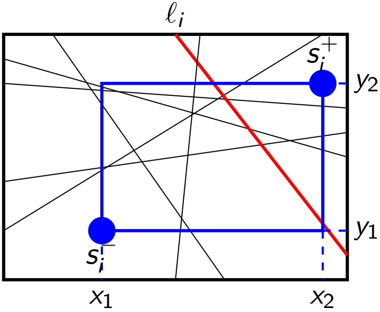

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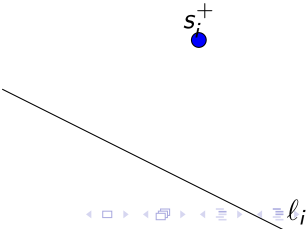

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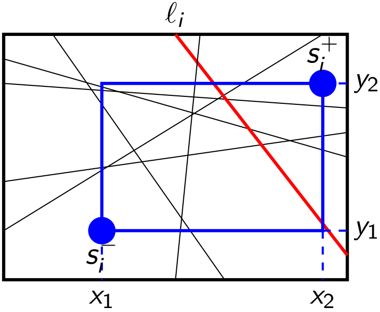

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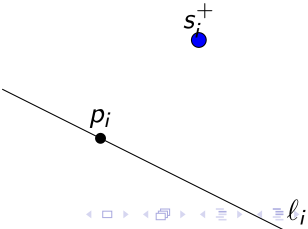

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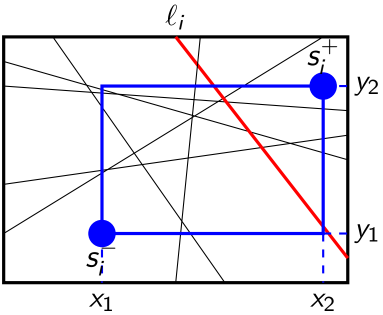

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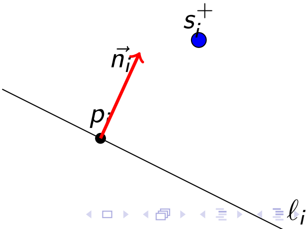

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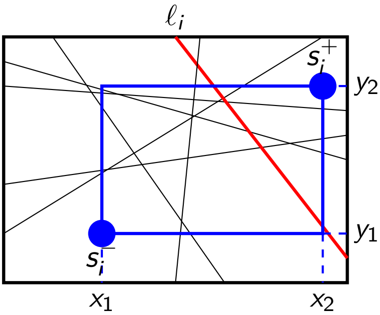

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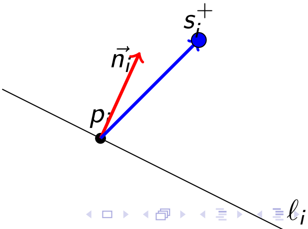

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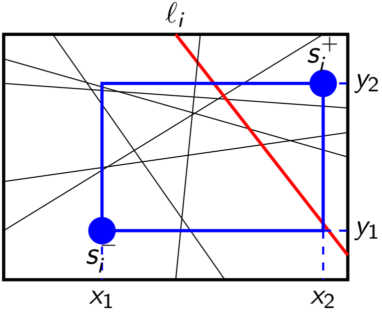

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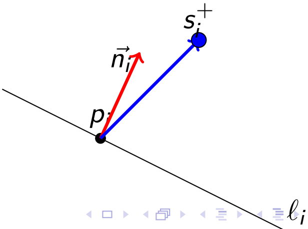

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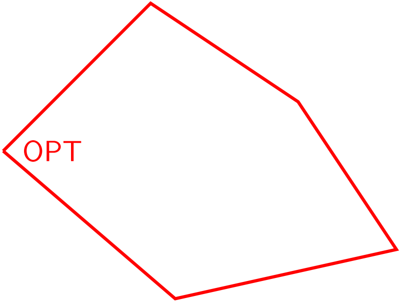
Analysis



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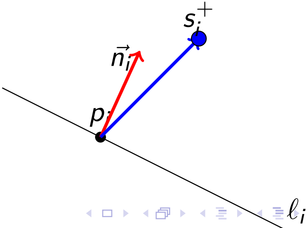

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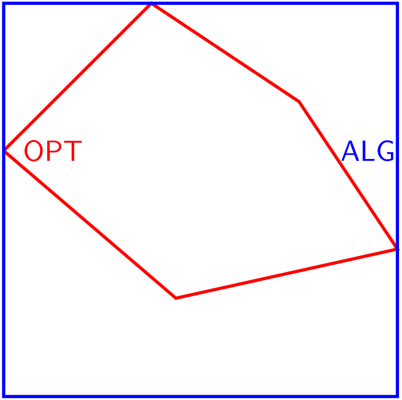

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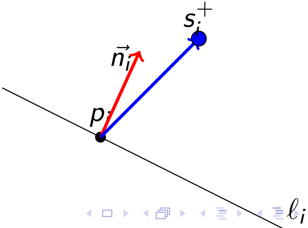

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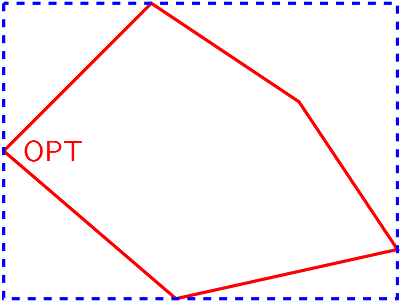

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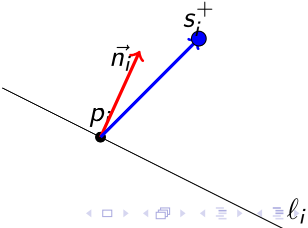

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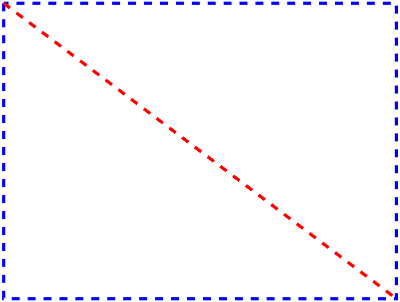

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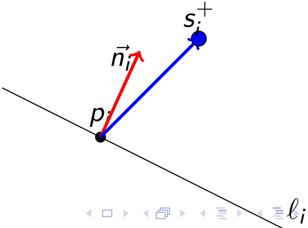

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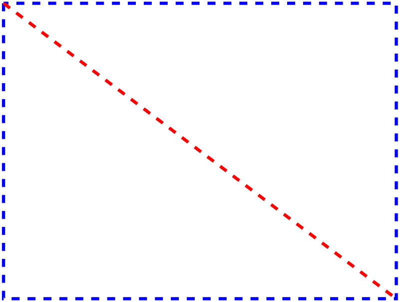

$$\frac{\text{cost}(\text{ALG})}{\text{cost}(\text{OPT})} \leq \frac{\text{perim}(\text{box})}{\text{diag}(\text{box})} \in 2^{O(d)}$$

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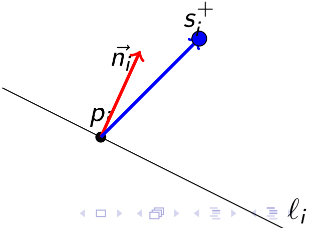

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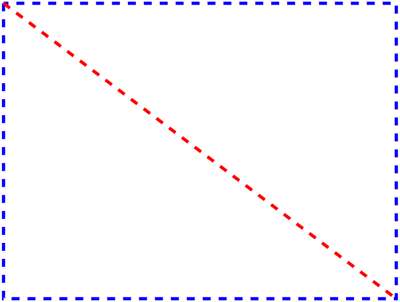

$$\frac{\text{cost(ALG)}}{\text{cost(OPT)}} \leq \frac{\text{perim(box)}}{\text{diag(box)}} \in 2^{O(d)}$$

Note: LP has constantly many variables $\to$ strongly polynomial linear time (Megiddo '84, Chan

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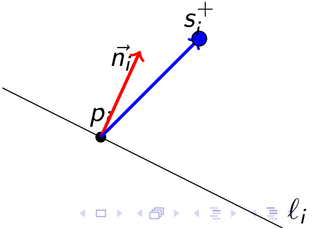

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Observation: Tour T is feasible $\Leftrightarrow$ conv(T) intersects all input hyperplanes.

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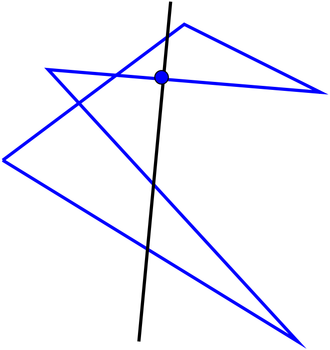

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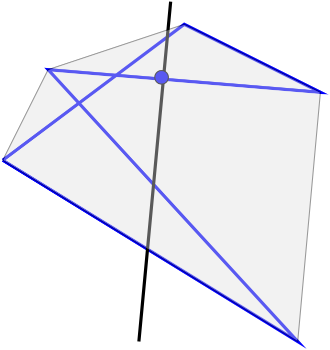

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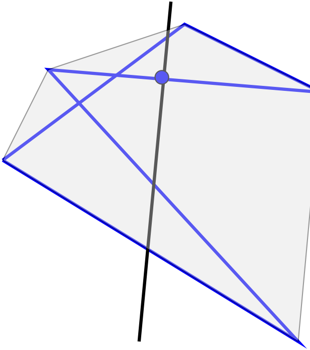

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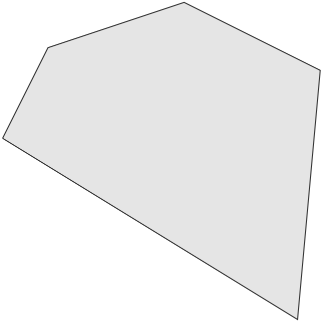

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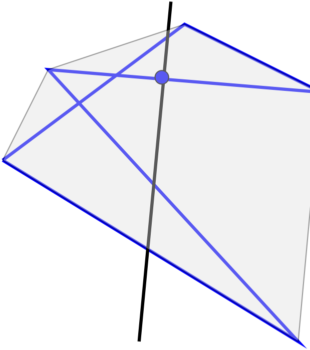

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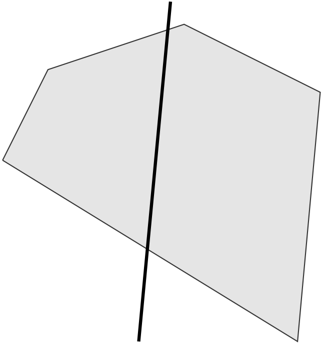

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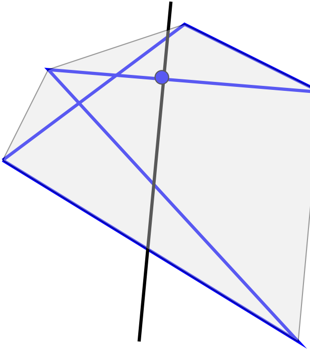

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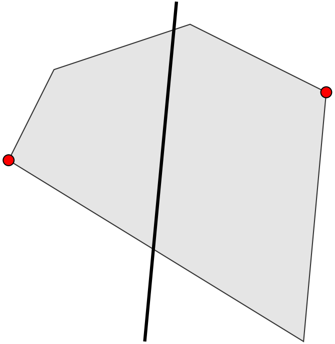

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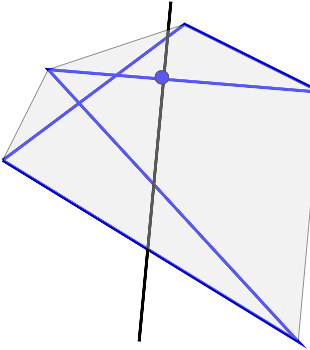

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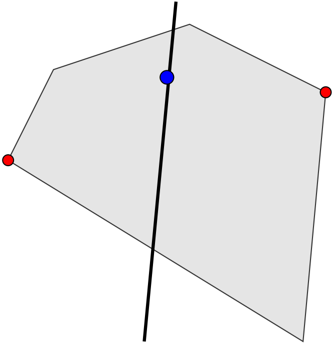

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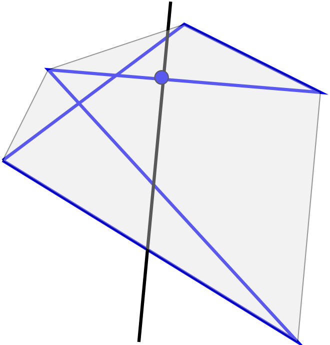

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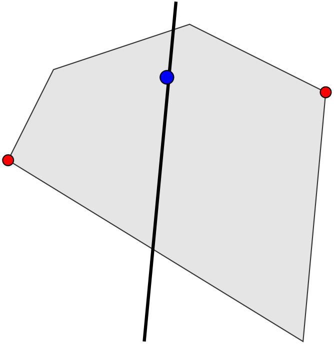

Shortest tour visiting all vertices in conv(OPT) is again optimal!

Roadmap for PTAS

- Define polytopes whose complexity is bounded by a function of ϵ (and d).

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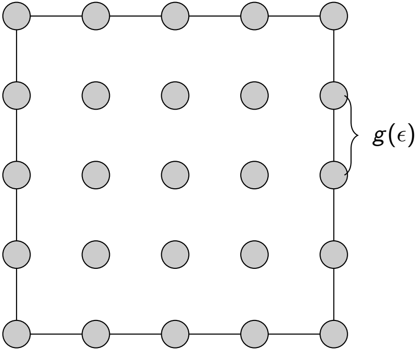
- Show that there is a $(1+\epsilon)$-approximation to OPT with a convex hull of bounded complexity.
- Use a linear program to find the "best" tour/polytope of bounded complexity.

Bounded Complexity Polytopes



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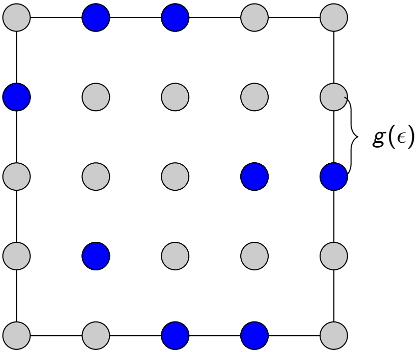
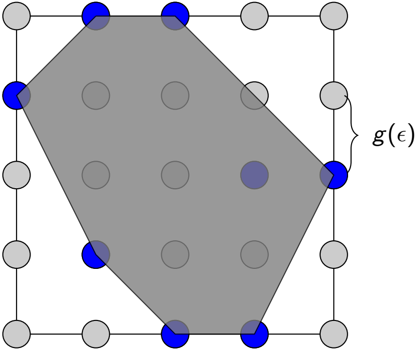


Bounded Complexity Polytopes



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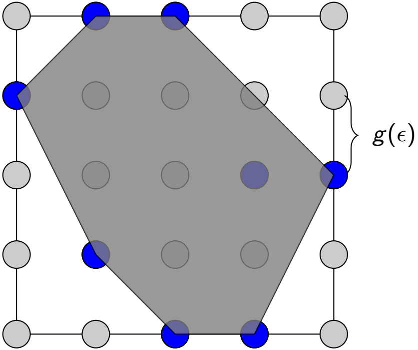


Scale & Translate

Bounded Complexity Polytopes suffice

Idea: Find a polytope P of bounded complexity so that $P \supseteq$ conv(OPT) and tsp(vertices(P))$\leq (1+\epsilon)\cdot$ cost(OPT).

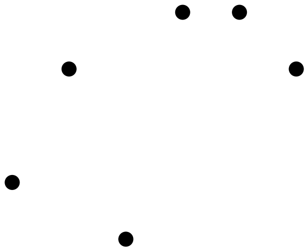
Two steps:

- **Sparcification** Find an intermediate polytope P' so that $P' \supseteq$ conv(OPT), tsp(vertices(P'))$\leq (1+\epsilon')\cdot$ cost(OPT), and P' has $O_{\epsilon,d}(1)$ many vertices.
- **Snapping:** Snap P' to the grid to obtain P, while increasing the tour length by at most another $(1+\epsilon'')$-factor.

Sparcification

Theorem (Chan '06)

Given an m-point set in $\mathbb{R}^d$*, one can construct an* ϵ*-core-set of size* $O(1/\epsilon^{(d-1)/2})$ *for the extent measure.*



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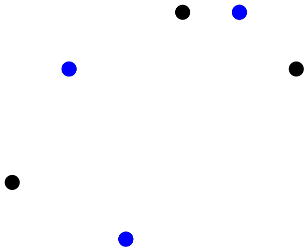

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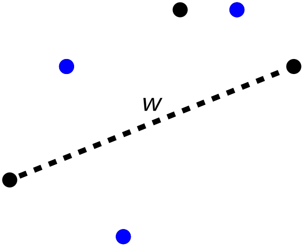

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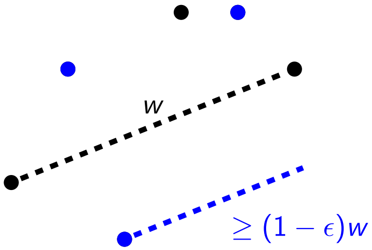

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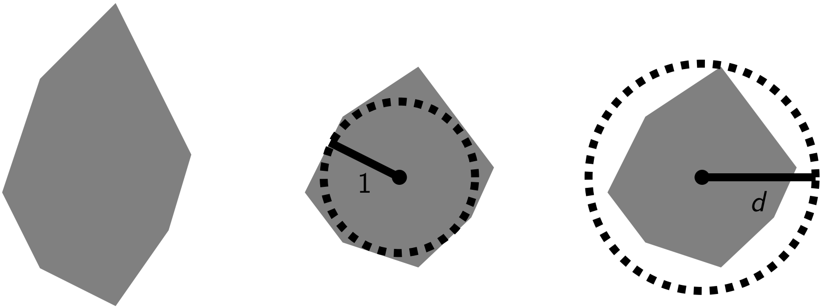
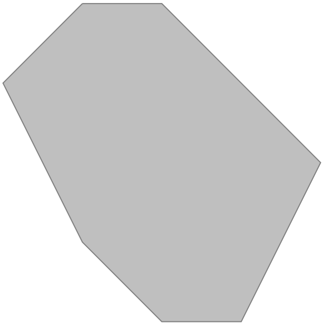


Sparcifying

Theorem

Theorem We can select $O_{\epsilon,d}(1)$ *many vertices of OPT such that their convex hull scaled by a factor* $(1+\epsilon')$ *contains OPT.*

Snapping



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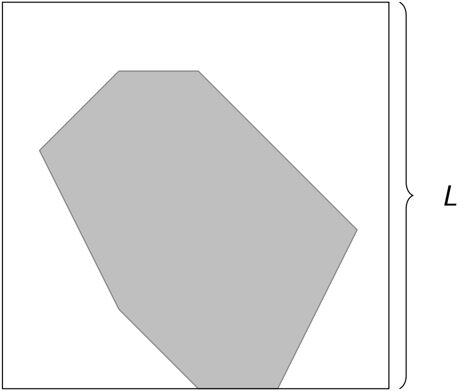

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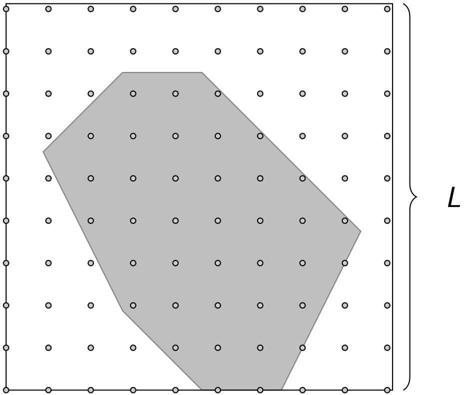

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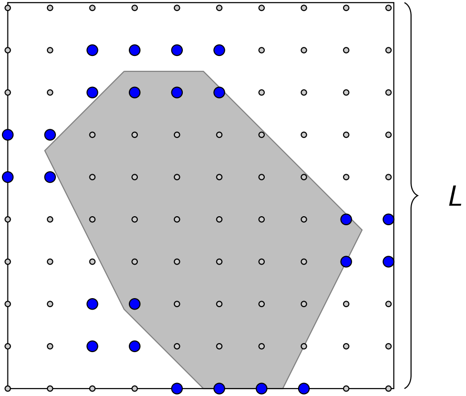

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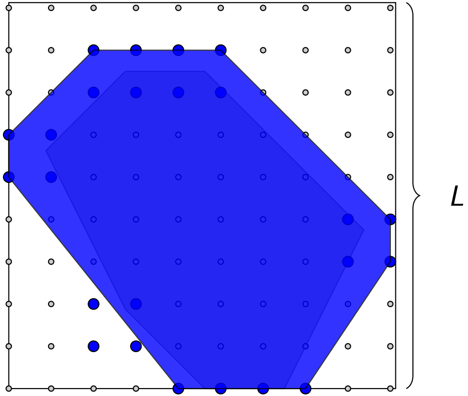

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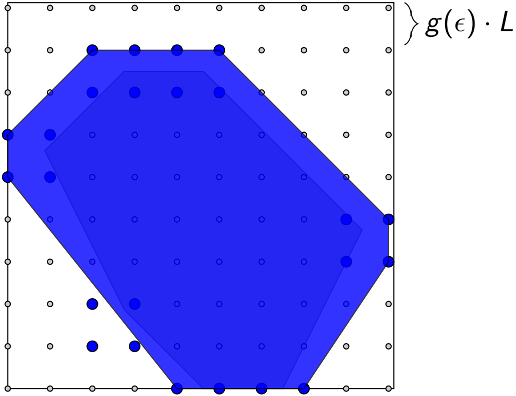

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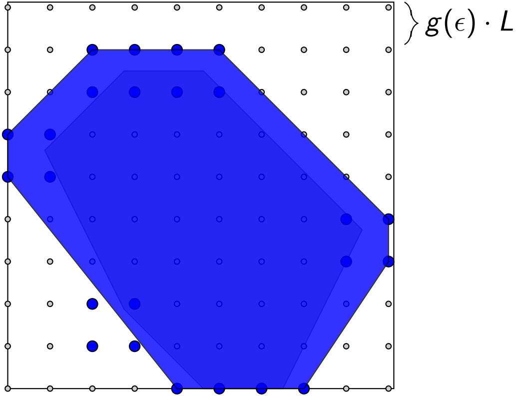

Note: $L \leq \text{cost(OPT)}$

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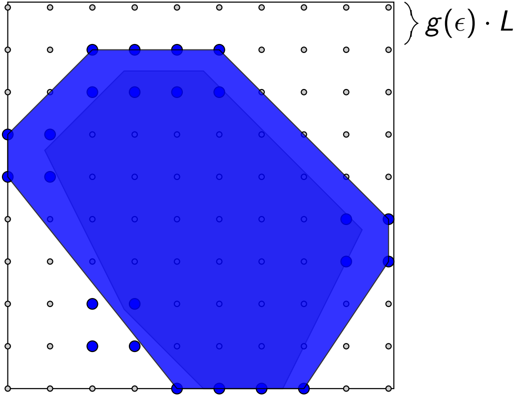

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We have:

- $P \supseteq P'$
- Detour cost: $O_\epsilon(1) \cdot 2^{O(d)} \cdot g(\epsilon) \cdot L \leq \epsilon \cdot \text{cost}(\text{OPT})$, for suitable $g(\epsilon)$.
- P is of bounded complexity.

Another Linear Program

"Guessing"

- The vertices of the polytope
- The order σ in which the tour (of length $\ell(\sigma)$) visits the vertices

Linear Program

- translation parameter $\vec{\rho}$
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LP has (again) constantly many variables $\rightarrow$ strongly polynomial linear time (Megiddo '84, Chan '06)

Summing up...

Theorem

TSP with hyperplane neighborhoods admits a PTAS with strongly polynomial linear running time.

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...still the constant is exponential in d and doubly exponential in $1/\epsilon$.

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TSP with hyperplane neighborhoods in $d = 2$, a.k.a. TSP with lines in 2d, can be solved exactly in polynomial time through an elegant reduction to the watchman route problem.

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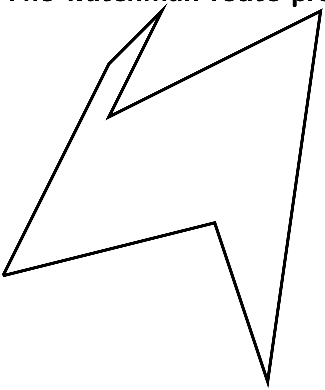

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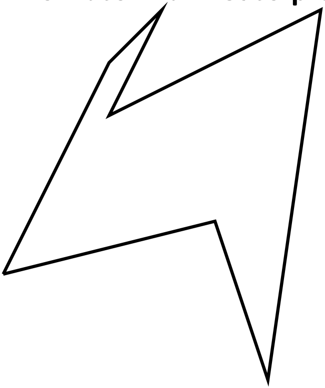

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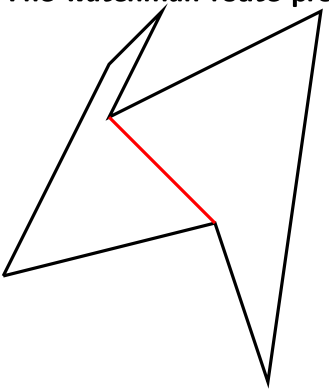

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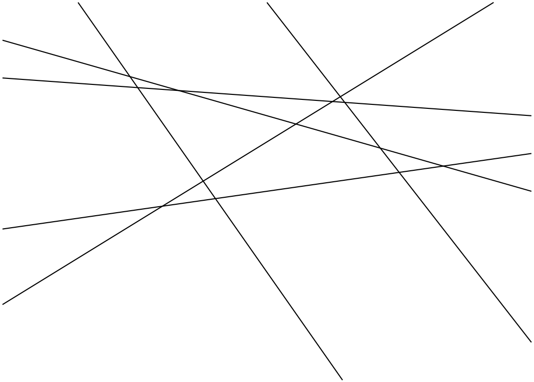

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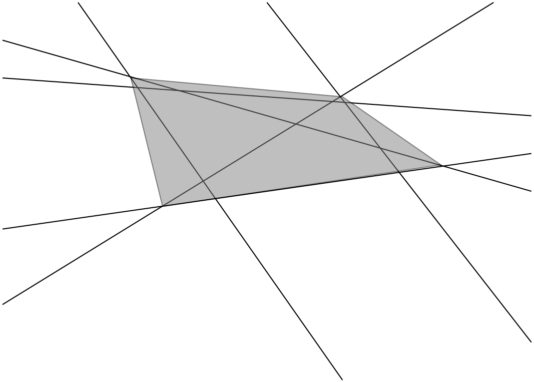

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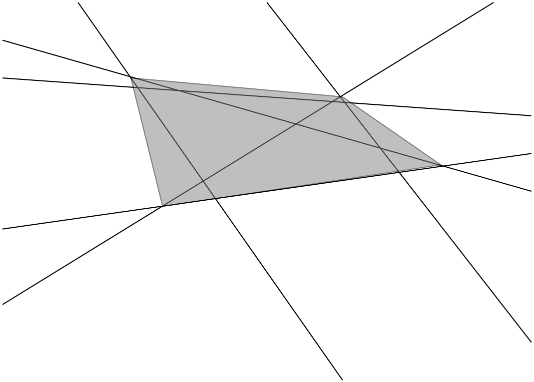

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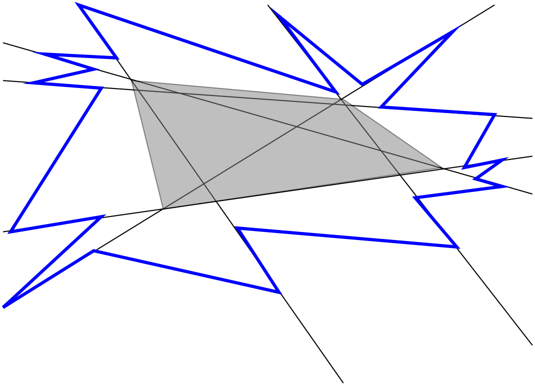

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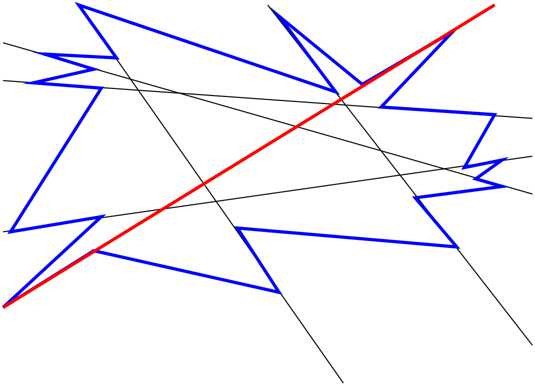

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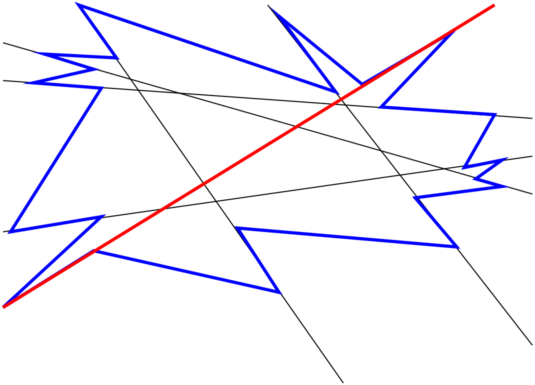

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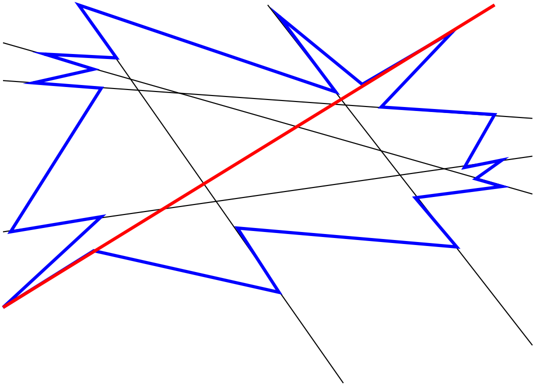

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The watchman route problem is solvable in polynomial-time (Tan '01)

Online Setting

Hyperplane Chasing (Definition)

Input: An initial point p_0 and a sequence of hyperplanes
Output: A sequence of points, one per hyperplane
Cost: Total moved distance

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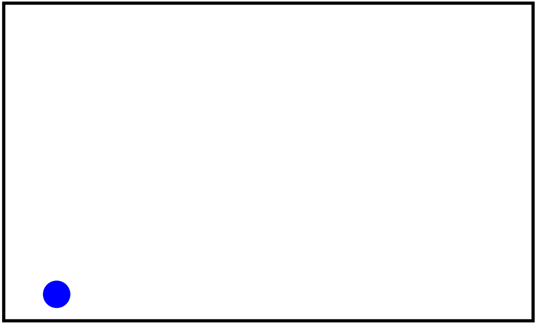

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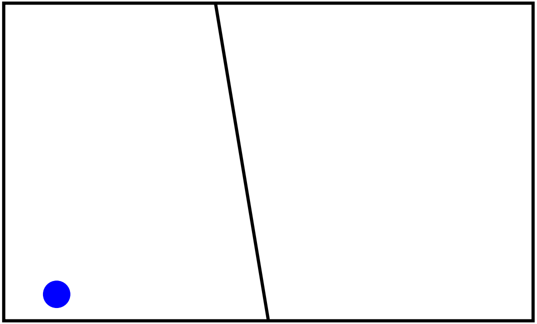

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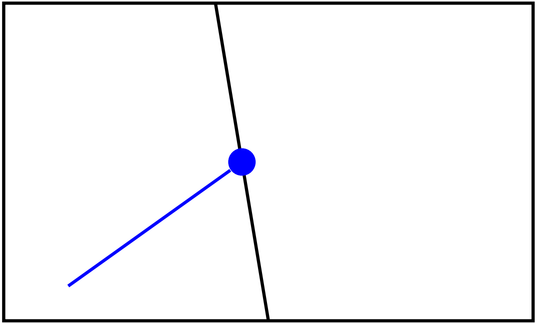

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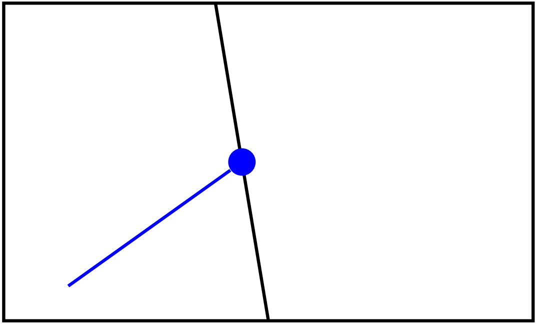

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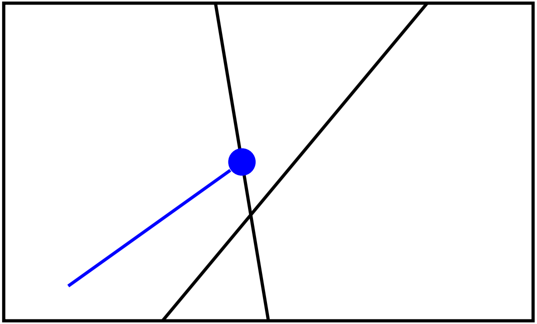

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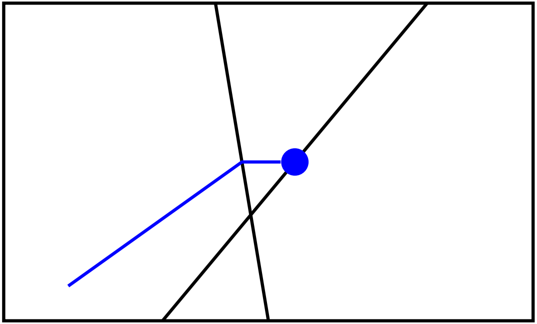

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Line Chasing for $d = 2$

Simple Algorithms:

Greedy:
$\omega(1)$-**competitive**

Move to intersection:
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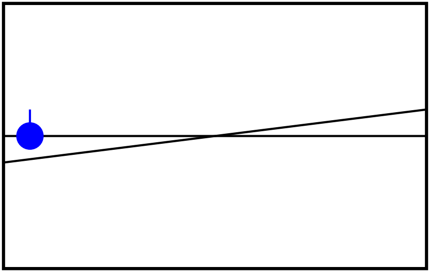

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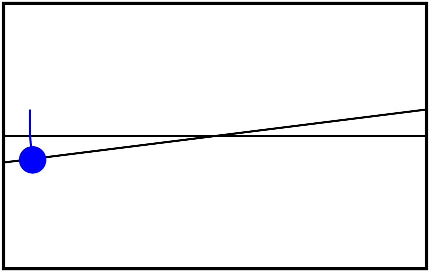

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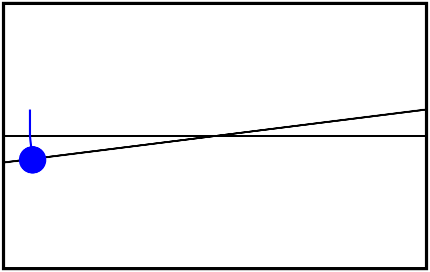

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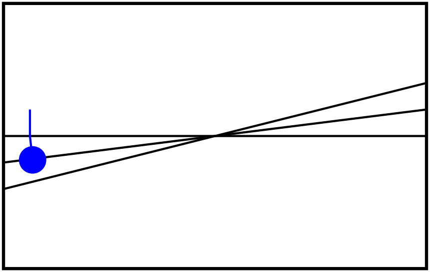

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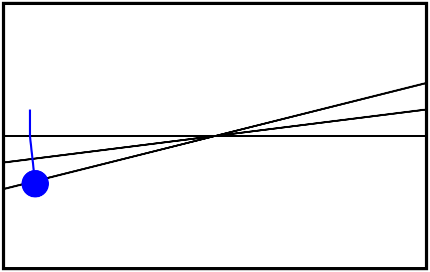

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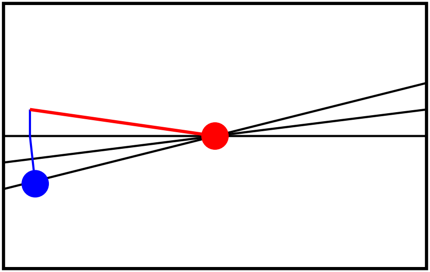

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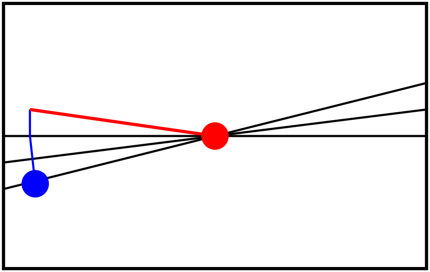

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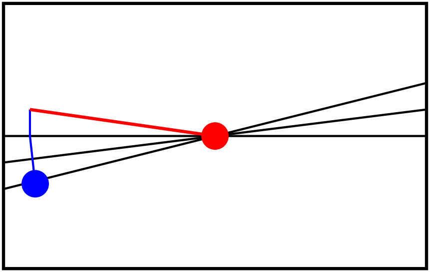

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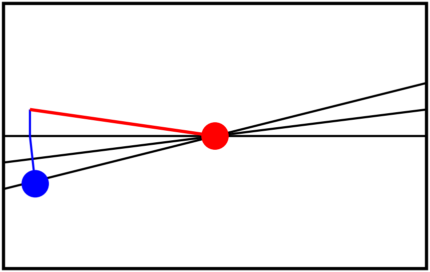

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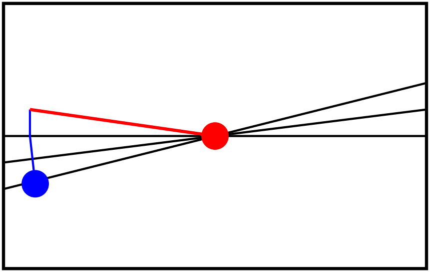

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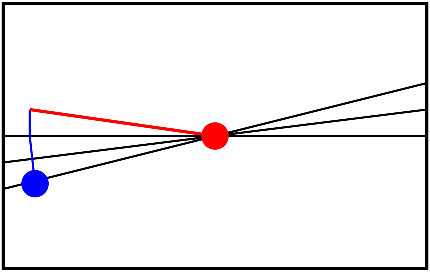

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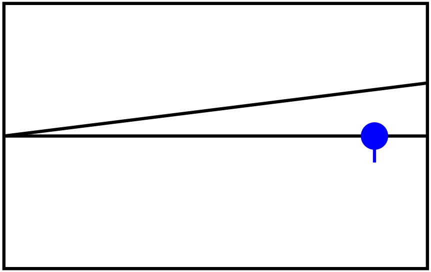

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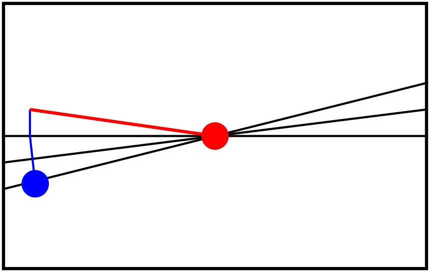

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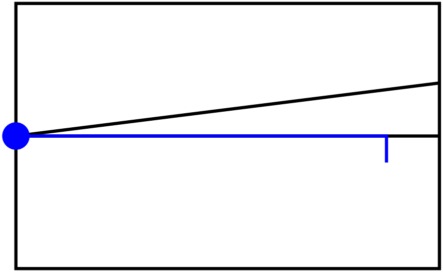

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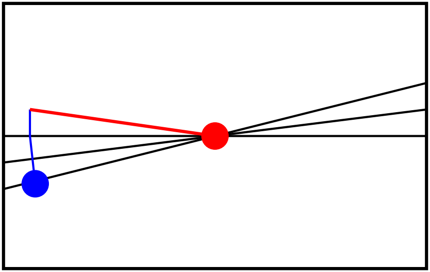

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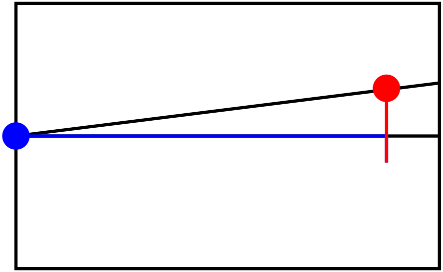

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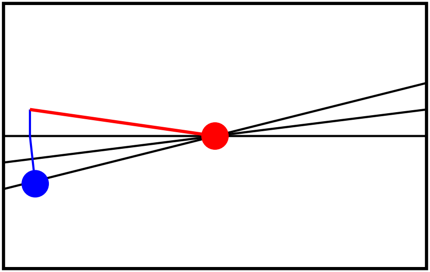
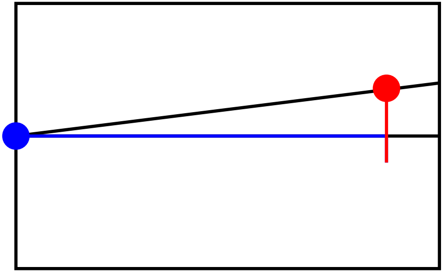


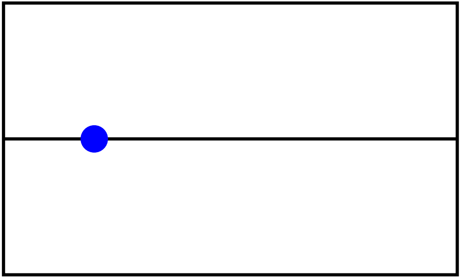
Move to intersection:
$\omega(1)$-**competitive**



Theorem (Friedman & Linial '93)

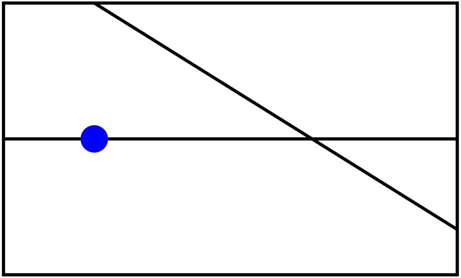
There is a $O(1)$-competitive algorithm for lines in $d = 2$.

Simpler $O(1)$-competitive algorithm (AA et al.'16)



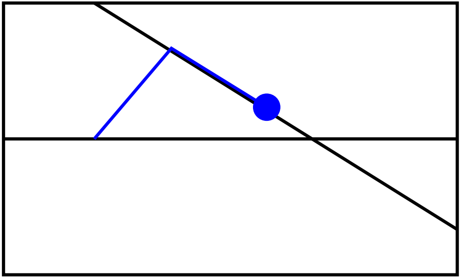
Algorithm:

Simpler $O(1)$-competitive algorithm (AA et al.'16)



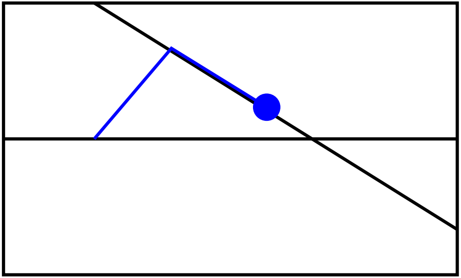
Algorithm:

Simpler $O(1)$-competitive algorithm (AA et al.'16)



Algorithm:
Move greedily to the next line, and then the same distance towards the intersection (if it exists).

Simpler $O(1)$-competitive algorithm (AA et al.'16)



Algorithm:
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Currently best algorithm: 3-competitive (Bienkowski et al. '18)

Convex Body Chasing

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- Simplified by Sellke & Argue, Gupta, Guruganesh and Tang, May '19

Future directions

What about:

- Is TSP with hyperplane neighborhoods NP-hard?
- Input hyperplanes have to be visited in a given order. Similar technique may work...
- Input is a set of lower dimensional affine subspaces. New techniques required...
- Best possible running time for $d = 2$?

Thanks!